A 10-bit S-box generated by Feistel construction from cellular automata

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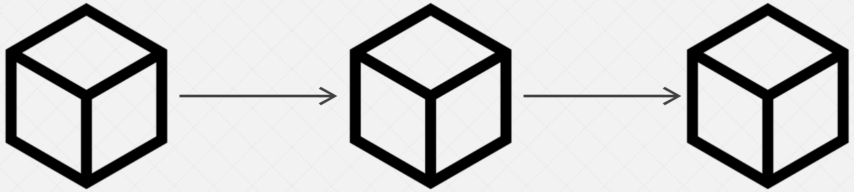
Block cipher encryption

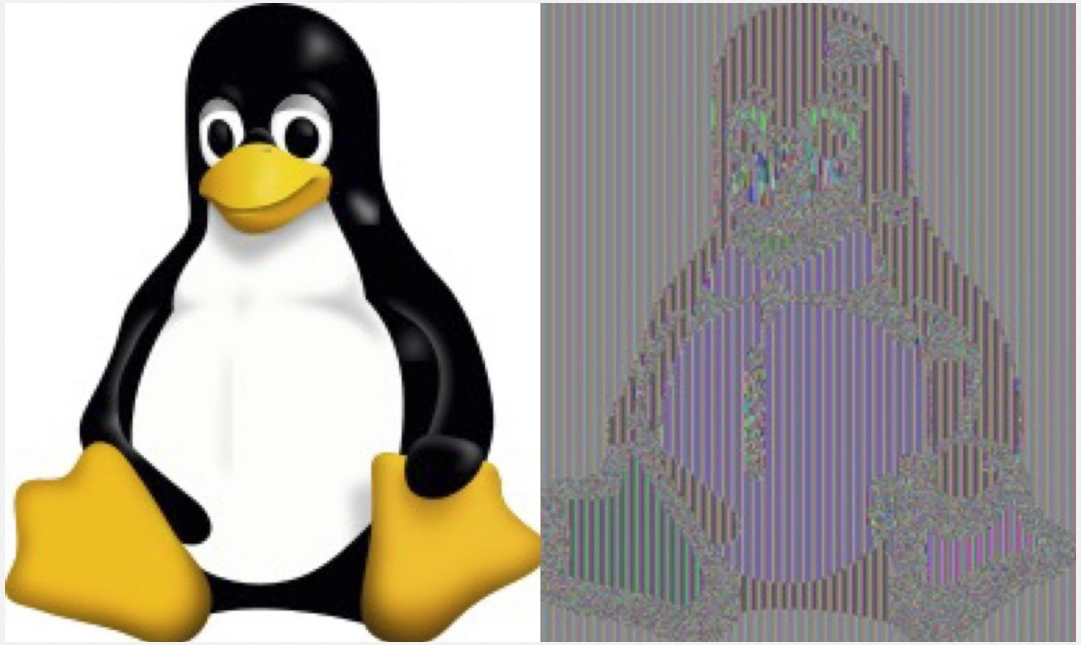
- Commonly used symmetric encryption
- Slicing the message into equal sized blocks



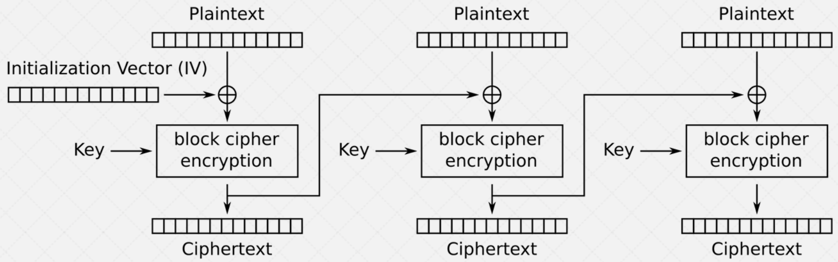
Example: **A**dvanced **E**ncryption **S**tandard (AES),
NIST standardized algorithm for symmetric cryptography

Blocks interdependecy

If each block was encrypted independently:

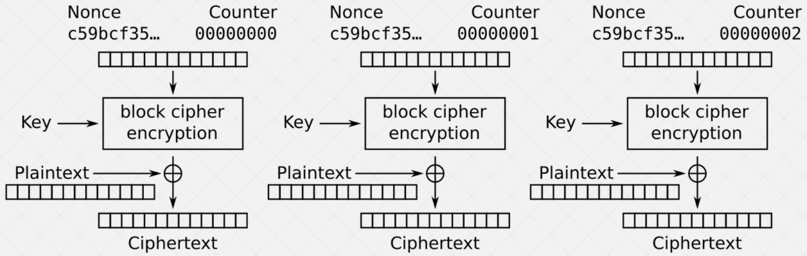


Solution 1: block chaining (CBC): not parallelisable



Cipher Block Chaining (CBC) mode encryption

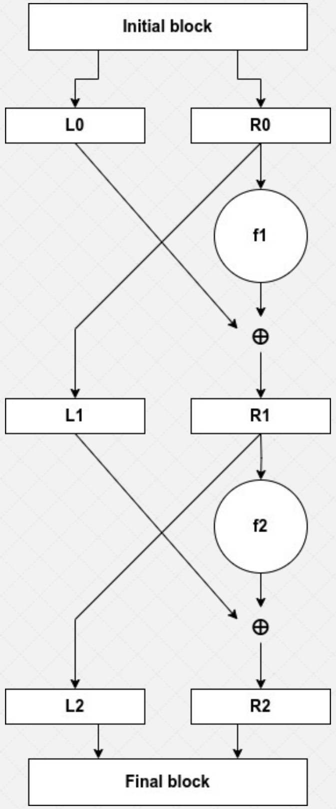
Solution 2: use a counter (GCM, CTR...)



Counter (CTR) mode encryption

Illustration of block encryption structure: Feistel networks

Used in some block cipher algorithms, like Blowfish
(AES uses another similar construction)



With:

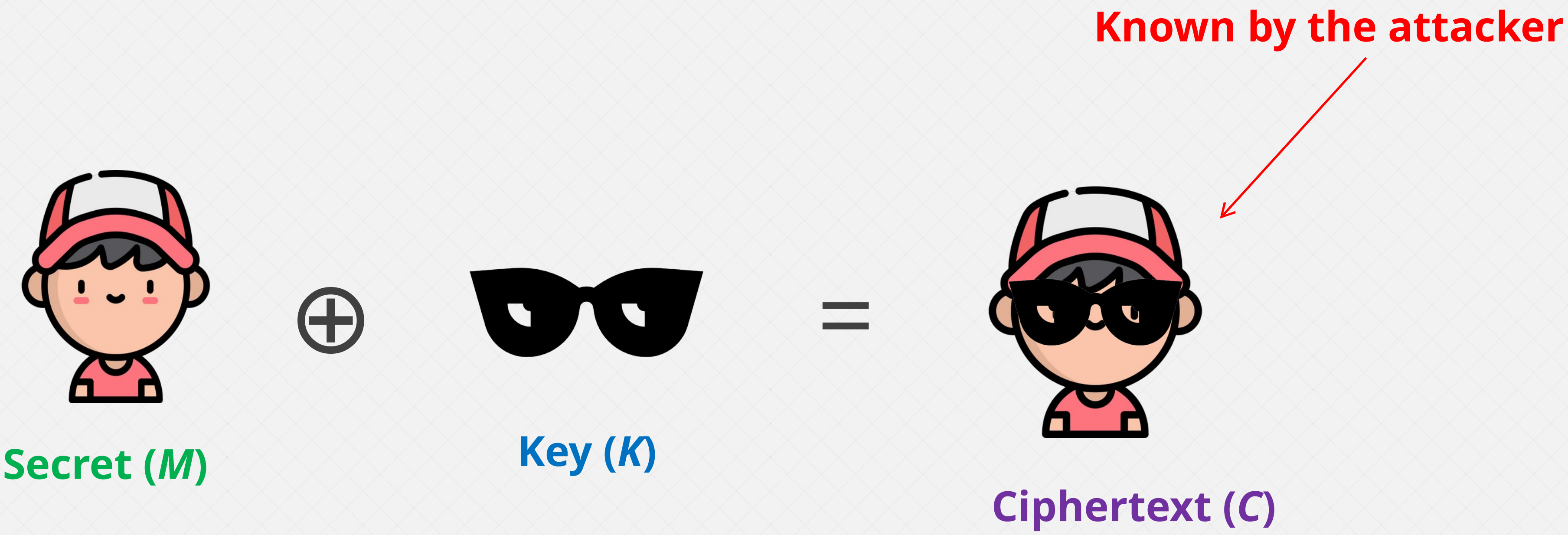
- *f*1 and *f*2: pseudo-random permutations
- ⊕ XOR operator (exclusive OR)
- Feistel network depth = 2

«pseudo-random» permutation:
Permutation that indistinguishable from a truly random permutation by a «*polynomial time adversary*» (an adversary with a computer with limited computing power)

But what are the subpermutations (*f*1, *f*2) made of?

Why do we need S-Boxes?

If block cipher was linear:



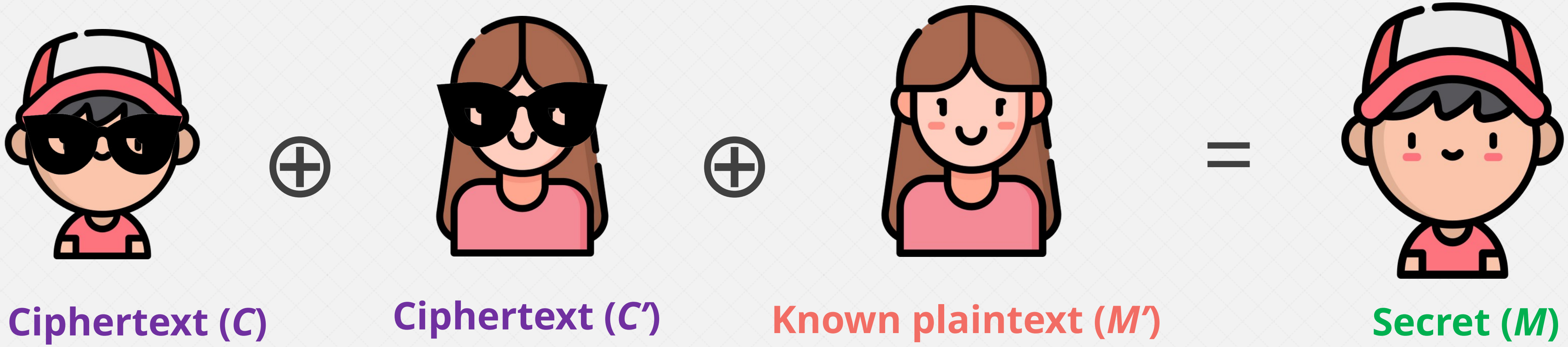
Why do we need S-Boxes?

If block cipher was linear:

Example of known plaintext: home page of bank website, before filling your credentials

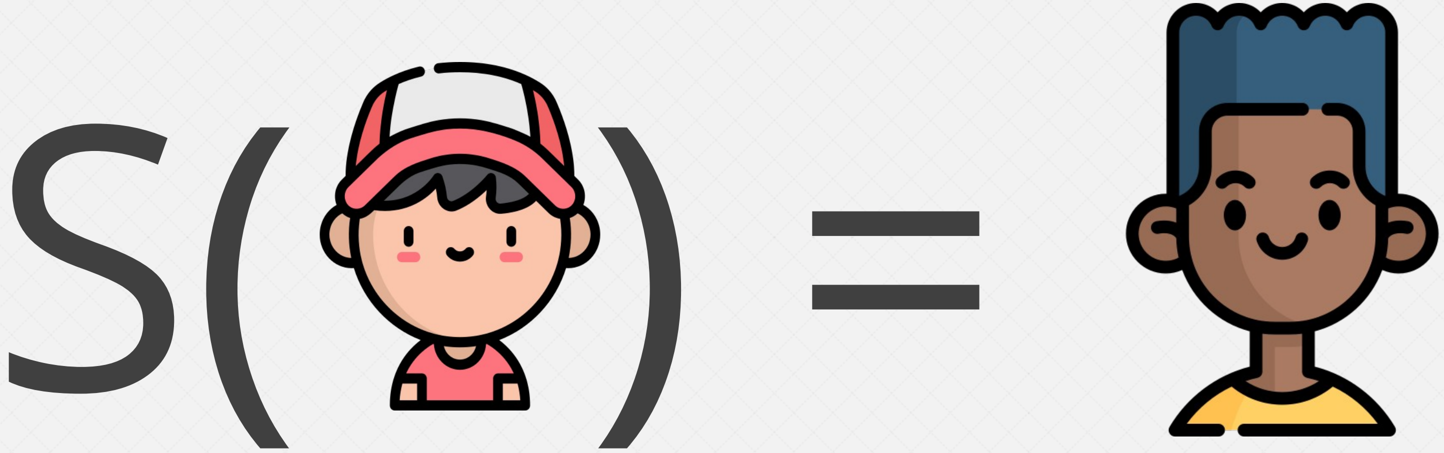
Why do we need S-Boxes?

If block cipher was linear:



This is a **known plaintext attack**

S-Box principle



So a simplified subpermutation round is **the S-Box action combined with a linear operation with the key**

A S-Box is a **public substitution table** that must be as far as possible from a linear function.
As we will see, there are other expected mathematical properties

S-Box example: PRESENT

x	0	1	2	3	4	4	6	7
S(x)	12	5	6	11	9	0	10	13

x	8	9	10	11	12	13	14	15
S(x)	3	14	15	8	4	7	1	2

A S-Box is a **public bijective* function $B^n \rightarrow B^n$ that is as far as possible from a linear function**

**There are non-bijective S-Boxes but this is not what we need here*

Boolean functions

$f(x_1, x_2, \ldots, x_n) = y$, with $x_1, x_2, \ldots, x_n, y \in \boldsymbol{B}$

Algebraic Normal Form (ANF):

$y = x_1*x_2*x_0 \oplus x_2*x_4 \oplus x_5 \oplus 1$

Here deg(f) = 3: size of the largest monomial

Linear function:

if degree = 1 ou degree = 0 (constant function)

There are $2^{\wedge(2^{\wedge}n)}$ possible n-variable Boolean functions

S-Box component functions

For $S(x_1, x_2, \ldots, x_n) = y_1, y_2, \ldots, y_n$, with $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in \boldsymbol{B}$

There are 2^n-1 component Boolean functions of S-Box S:

- $f_1(x_1, x_2, \ldots, x_n) = y_1$
- $f_2(x_1, x_2, \ldots, x_n) = y_2$
- …
- $f_{n+1}(x_1, x_2, \ldots, x_n) = y_1 \oplus y_2$
- …
- $f_{2^\wedge n\text{-}1+1}(x_1, x_2, \ldots, x_n) = y_1 \oplus y_2 \oplus \ldots \oplus y_n$

S-Box component functions

Example:

For *S* defined as:

x	00	01	10	11
$S(x)$	10	00	11	01

We have:

x	$f_1(x) = y_1$
00	1
01	0
10	1
11	0

x	$f_2(x) = y_2$
00	0
01	0
10	1
11	1

x	$f_2(x) = y_1 \oplus y_2$
00	1
01	0
10	0
11	1

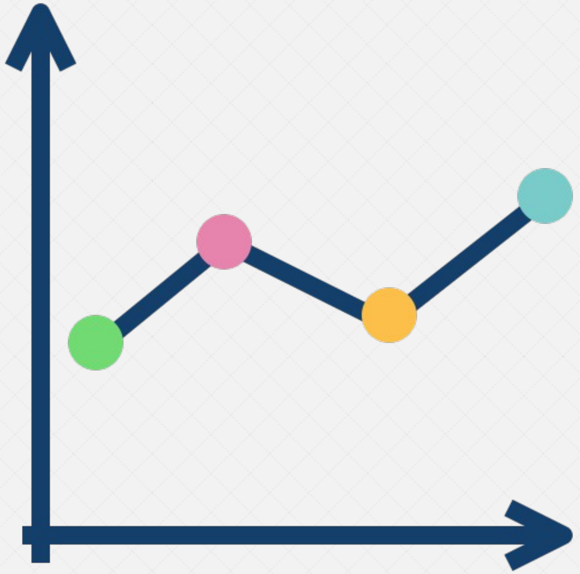
S-Box Mathematical properties

Exhaustive list:

- Min and max algebraic degree
- Algebraic complexity
- Nonlinearity
- Strict Avalanche Criterion (SAC)
- Bit Independence Criterion (BIC)
- Linear Approximation Probability (LAP)
- Differential Approximation Probability (DAP)
- Differential Uniformity (DU)
- Boomerang Uniformity (BU)

Nonlinearity

- For each component function, number of bits that should be switched to have a linear function
- The worst value is the metric



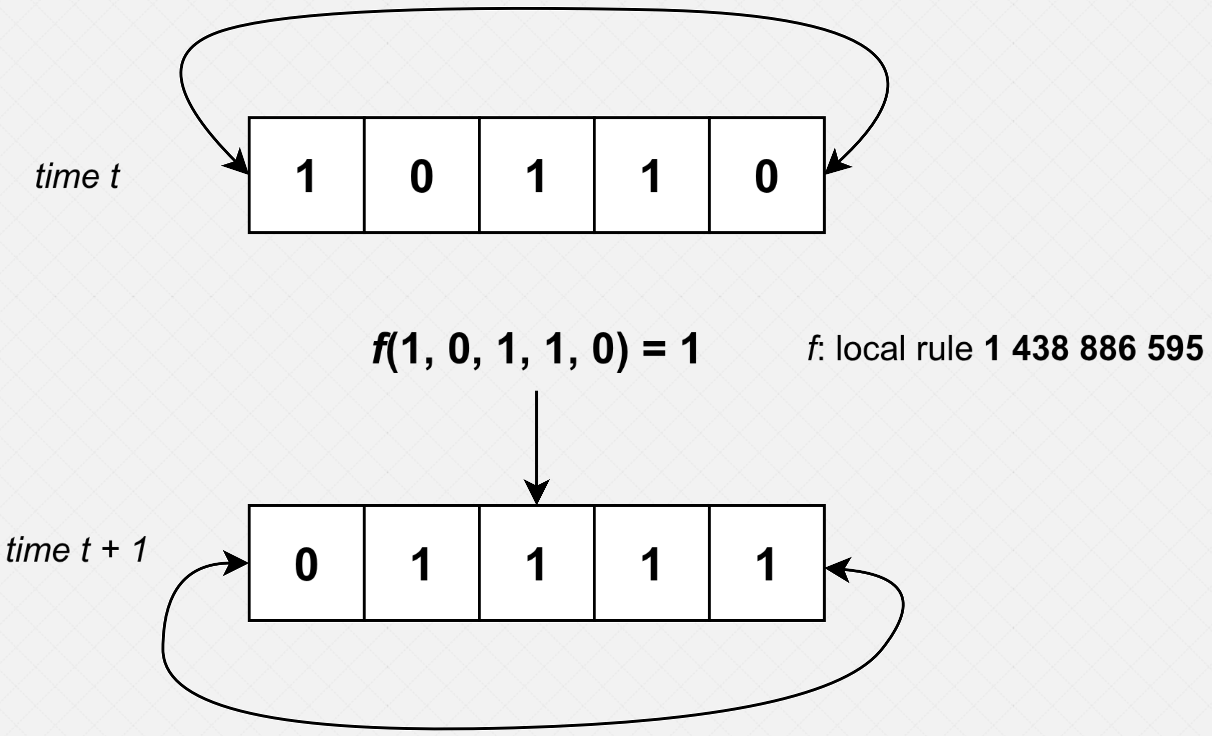
- A **high value enables linear cryptanalysis resistance**

Bit Independence Criterion

- BIC is satisfied when for all input bit *k*, for all output bits *i*, *j*, flipping k^{th} input bit flips i^{th} and j^{th} output bits independently

- The metric is a number between 0 and 1 (closest to satisfy the BIC), **1 the worst and 0 the best**

Uniform cellular automaton



- Ring* of Boolean cells
- At each **discrete** time step, each cell is updated according to its value and the values of its neighbors, according to a weel chosen **local transition function**

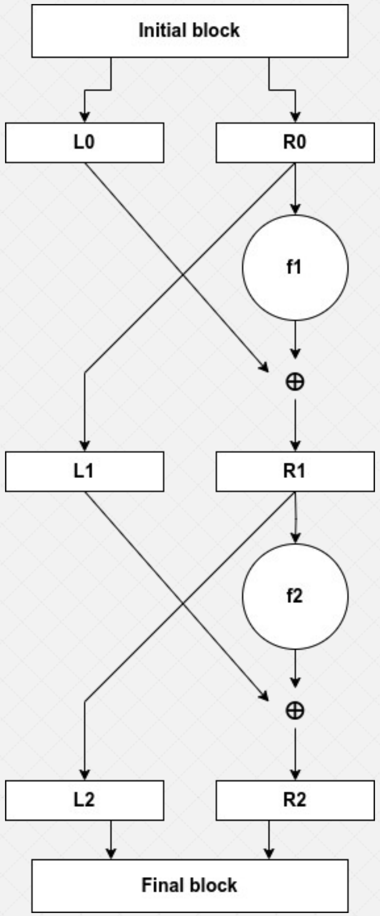
In this specific case

With $f(x) = x_0{*}x_3 \oplus x_1{*}x_3 \oplus x_1 \oplus x_2{*}x_3 \oplus x_2 \oplus x_3{*}x_4 \oplus x_3 \oplus 1$

1 438 886 595 is the **decimal representation** of the truth table

Construction of our 10-bit S-Box

Our S-Box itself is a **sub 10-bit Feistel network**, of depth 11



Empirical construction based on cryptanalysis:

- f_1: affine function: f(x) = 5x+3 mod 31
- f_2 to f_5: 1 generation of our automaton
- f_6: affine function: f(x) = 7x+11 mod 31
- f_7 to f_9: 1 generation of our automaton
- f_{10}: affine function: f(x) = 13x+17 mod 31
- f_{11}: 1 generation of our automaton

Results

Comparison with AES S-Box *(values are normalized to compare a 10-bit S-Box with a 8-bit S-Box)*

Property	Our 10-bit S-Box	8-bit AES S-Box
Min algebraic degree	**8**	7
Max algebraic degree	**9**	7
Algebraic complexity	**1023**	255
Nonlinearity	**434 (= 108.5 * 4)**	112
Strict Avalanche Criterion	**0.44 - 0.5 - 0.57**	0.45 - 0.5 - 0.56

Results

Comparison with AES S-Box *(values are normalized to compare a 10-bit S-Box with a 8-bit S-Box)*

Property	Our 10-bit S-Box	8-bit AES S-Box
Bit Independence Citerion	**0.124**	0.134
Linear Approximation Probability	**9.28%**	6.25%
Differential Approximation Probability	**1.37%**	1.56%
Differential Uniformity	**14**	4
Boomerang Uniformity	**24**	6

THANK YOU

Questions ?

Min and max algebraic degree

Size of the largest monomial of each function:

- If $f1(x_1, x_2, \ldots, x_n) = x_1 * x_2 * x_4 \oplus x_1 * x_2 \oplus x_3$ then deg($f1$) = 3
- Largest and lowest degree of each component function

Large values avoid «Low order approximation attack»

Strict avalanche criterion

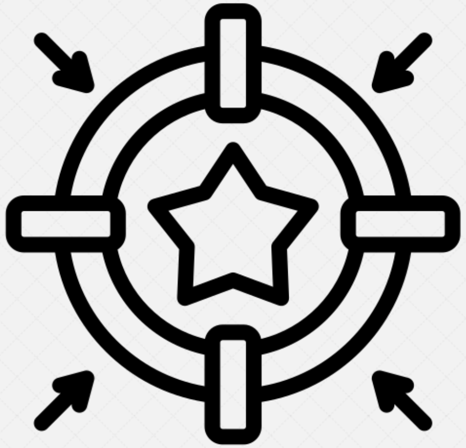
- When an input bit is flipped, 50% of the output bits must be flipped on average
- The ideal value is 50%

We define a table of size *n***n*:

- When the i^{th} input bit is flipped, in which proportion is the j^{th} output bit flipped?

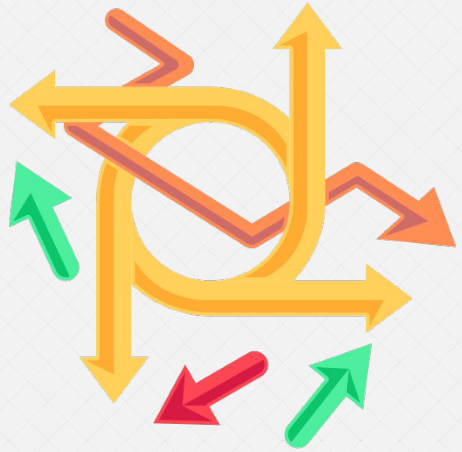
Each table value should be as close as possible of 50%

Differential uniformity



- Gives proximity to a perfectly nonlinear S-Box (impossible for bijectivity)
- For each combination (a, b), differential uniformity table δ gives the number of inputs x such that $S(x) \oplus S(x \oplus a) = b$
- The metric is then $U = \max(\delta)$
- The **lowest value is the best**

Algebraic complexity



Our S-Box is represented over ℕ:

$S(x) = a_0 + a_1 * x + \ldots + a_{(2^n)-1} * x^{(2^n)-1}$ mod 2^n avec $x, a_0, a_1, \ldots \in [\![0, 2^n-1]\!]$

Algebraic complexity is the number of monomials in the univariate polynomial

A **large value protects against interpolation attacks**

Linear Approximation probability

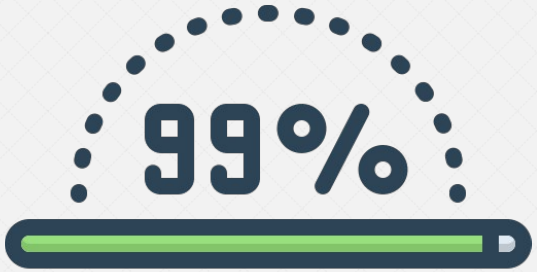
- Gives an indication about S-Box resistance against linear cryptanalysis
- Defined as the maximum correlation between *α*x* et *β*S(x)*, pour tout *α* et *β* $\in [\![1, 2^n]\!]$
- **Lowest value is the best**

Differential Approximation probability

Given by the XOR distribution between input and output

- For each combination *(Δx, Δy)*, differential probability table DP gives the number of inputs *x* such that $S(x) \oplus S(x \oplus \Delta x) = \Delta y$
- So DAP = max(DP)

A **low value ensures resistance** against differential cryptanalysis



Boomerang Uniformity

- Defines S-Box resistance against boomerang attacks (a variant of differential cryptanalysis)
- For each combination *(a, b)*, Boomerang Connectivity Table (BCT) gives the number of inputs *x* such that:

$$S^{-1}(S(x) \oplus b) \oplus S^{-1}(S(x \oplus a) \oplus b) = a$$

- *BU* = max(BCT)
- The **lowest value is the best** against boomerang attacks